Using controlling chaos technique to suppress self-modulation in a delayed feedback traveling wave tube oscillator

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Controlling chaos

An idea of controlling chaos technique for stabilizing of unstable periodic orbits in dynamical systems was suggested by Ott, Grebogy and Yorke (PRL 64, No. 11. P. 1196-1199 (1990))


\[ \frac{dx}{dt} = F[x(t)] \] — dynamical system with chaotic dynamics

\[ \frac{dx}{dt} = F[x(t)] + \varepsilon (x(t) - x(t-T)) \] — system with time-delayed control with delay time equal to period of motion to be stabilized

Usually \( T \) is unknown a priori
Does not allow to stabilize high-frequency motion

A.M. Dolov and S.P. Kuznetsov (Tech. Phys. 73, No. 8. P. 139-142 (2003)) — suppress self-modulation in a microwave vacuum tube oscillator via modulation of electron beam current by external feedback control signal with delay time which depends on self-modulation period.
Delayed feedback oscillator
Method of chaos control

\[ A_{in}(t) = \rho (1-k) A_{out}(t - \tau_1) e^{i\psi_1} + \rho k A_{out}(t - \tau_2) e^{i\psi_2} \]

- \( k \) — control parameter,
- \( \rho \) — attenuation produced by the VA

**Idea:** To choose delay times and phases so that fundamental waves passing through two feedback legs appear in same phase, while the self-modulational sidebands appear in anti-phase and suppress each other.
Method of chaos control

Consider propagation of a modulated signal

\[ A(t) = [A^{(\omega)}(t) + A^{(\omega+\Omega)}(t)e^{i\Omega t} + A^{(\omega-\Omega)}(t)e^{-i\Omega t}]e^{i\omega t} \]

Substituting into the boundary condition one can show that if we adjust the parameters as

\[ \psi_2 - \psi_1 - \omega(\tau_2 - \tau_1) = 2\pi n \]

\[ \Omega(\tau_1 - \tau_2) = 2\pi m + \pi \]

we obtain

\[ A_{in}^{(\omega)} = \rho [1 - k + k]e^{i(\psi_1 - \omega \delta_1)} A_{out}^{(\omega)} = \rho e^{i(\psi_1 - \omega \delta_1)} A_{out}^{(\omega)} \]

— same as for the oscillator with single feedback. **Non-invasive control.**

and

\[ A_{in}^{(\omega\pm\Omega)} = \rho (1 - 2k)e^{i(\psi_1 - (\omega\pm\Omega)\delta_1)} A_{out}^{(\omega\pm\Omega)} \]

Sideband waves coming from different feedback legs weaken and for \( k=1/2 \) completely suppress each other.
Delayed feedback oscillator with cubic nonlinearity

\[
\frac{dA}{dt} + \gamma A = \alpha \left[ (1-k)(1-|A(t-\tau_1)|^2)A(t-\tau_1)e^{i\psi_1} + k(1-|A(t-\tau_2)|^2)A(t-\tau_2)e^{i\psi_2} \right]
\]

Nonlinear dynamics of a single-feedback oscillator \((k=0)\) was studied in details in N.M. Ryskin, A.M. Shigaev. Complex Dynamics of a Simple Distributed Self–Oscillatory Model System with Delay, Technical Physics 47, 795-802 (2002).

\[
\begin{align*}
\psi_1 &= 0.01\pi \\
\gamma &= 1.5 \\
\tau_1 &= 1 \\
\alpha &= 3.6 \\
\Omega &\approx 0.7\pi
\end{align*}
\]

With the increase of \(\alpha\) — self-excitation \(\rightarrow\) single-frequency generation \(\rightarrow\) self-modulation \(\rightarrow\) period doublings \(\rightarrow\) chaos
Adding of the secondary control feedback allows to suppress self-modulation

According to derivations presented above we must choose $\tau_2=2.43$, $\psi_2=2.02\pi$

Adding of the secondary control feedback allows to suppress self-modulation
Suppressing of self-modulation

$k=0$

$k=0.2$

Same for deep self modulation after period doubling bifurcation, $\alpha=4.3$
Map of dynamic regimes on $k$–$a$ plane

(1) — no generation, (2) — single-frequency generation, (3) — periodic self-modulation, (4) — chaos

One can see that the method works only for $k < 0.3$
This is caused by excitation of another sideband mode

Fundamental frequency does not depend on $k$ while the modulation frequency switches to the frequency of another mode at $k \approx 0.3$
Simple 4D map model (limit $\gamma >> 1$)

$$A_{n+1} = \frac{\alpha}{\gamma} \left[ (1 - k) \left( 1 - |A_n|^2 \right) A_n e^{i\psi_1} + k \left( 1 - |A_{n-1}|^2 \right) A_{n-1} e^{i\psi_2} \right]$$

4-th order characteristic equation allows factorization in 2 second-order equations that are easy to solve analytically

$$\mu^2 + \mu \left( (1 - k) \left( 2 - \frac{\alpha}{\gamma} \right) \pm k \left( 1 - \frac{\alpha}{\gamma} \right) \right) - \left( k \left( 2 - \frac{\alpha}{\gamma} \right) \pm (1 - k) \left( 1 - \frac{\alpha}{\gamma} \right) \right) = 0$$

$$\frac{\alpha}{\gamma} = \frac{3}{2} + \frac{1}{2(1-2k)} \quad \text{Threshold of PD bifurcation, } \mu = -1$$

$$\frac{\alpha}{\gamma} = \frac{3}{2} + \frac{1}{2k} \quad \text{Threshold of Neimark–Sacker bifurcation, } \mu = \exp(i\theta)$$

$$\cos \theta = \frac{k - 1}{2k} \quad \text{Winding number (} k > 1/3 \text{)}$$
Sensibility to mismatch of the delay parameter $\tau_2$
Nonlinear Schrödinger equation with delayed boundary condition

\[ i \left( A_t + V_g A_x \right) + \frac{\omega''}{2} A_{xx} + \beta |A|^2 A = 0 \]

Nonlinear dynamics of the single-feedback \((k=0)\) system has been studied in details in A.A. Balyakin, N.M. Ryskin, O.S. Khavroshin, (Radiophys. Quant. Electron., 2007, to be publ.).
Modified Ikeda map (zero dispersion limit)

\[ A_{n+1} = A_0 + (\rho - \rho_1) A_n e^{i(\varphi + |A_n|^2)} + \rho_1 A_{n-1} e^{i(\varphi + |A_{n-1}|^2)} \]

Characteristic equation

\[ \mu^4 - 2\mu^2 \rho (\mu (1 - k) + k) (\cos \Phi - i \sin \Phi) + \rho^2 (\mu (1 - k) + k)^2 = 0 \]

\[ \Phi = \varphi + |A|^2 \]

Analytical expressions for PD (solid) tangent (dashes) and Neimark–Sacker (dots) bifurcations were obtained

\[ \cos \theta = \frac{k - 1}{2k} \]

The same equation for the winding number \( k > 1/3 \)
Numerical results for the modified Ikeda map

Bifurcation maps on $A_0$-$\phi$ plane ($\rho=0.5$). One can see excellent agreement with analytic theory. 1 — period 1 motion, 2 — period 2 motion, ..., $Ch$ — chaos.
Numerical results for the modified Ikeda map

$k = 0.24$
Numerical results for the modified Ikeda map

With the increase of $k$ boundary of the Ikeda instability shifts up
However, for $k>1/3$ domain of quasi-periodic motion appears above the line of Neimark–Sacker bifurcation. Thus, the control is most effective for $k=1/3$.  

Numerical results for the modified Ikeda map

$k = 0.36$
Numerical results for NLS with delayed feedback

Parameters: \( V=1, \beta=1, \omega''_0=0.01, \rho=0.5, \psi_1=0, L=5, T_1=10, \omega=0, A_0=0.45 \)

Without control \((k=0)\) deep self-modulation with period \( T_{sm} \approx 40 \) is observed

With control \((k=0.2, T_2=30, \psi_2=0)\) we get stable single frequency oscillations

\[
\psi_1 - \psi_2 - \omega (T_1 - T_2) = 2\pi n
\]

\[
\Omega(T_1 - T_2) = 2\pi m + \pi
\]
Numerical results for NLS with delayed feedback

Same for the case of strong dispersion ($\omega_0'' = 1$).

Now self modulation is caused by modulation instability, not by Ikeda instability (see our paper to be publ. in Radiophys. Quant. El., 2007 for details). Completely different self-modulation period, $T_{sm} \approx 10$.

Thus we need to change the control feedback delay, $T_2 = 5$. 

$A_0 = 0.22$
Summary

The proposed modification of time-delayed auto-synchronization technique for controlling chaos allows suppressing various instabilities in systems with time-delayed feedback. The method is useful to provide stable single frequency oscillations in various RF, microwave and optical devices.